

MTH401 – Differential Equations – Final term Paper

Session 1 – Spring

M = Marks= 53
Time: 120min

MTH401 - Differential Equations - Q. No. 1 (M - 2)

If the variation of the path of the curves can be described by the concept of differential Equations, then which of the following differential equation describe the path for y-axis

- ▶ $\frac{dy}{dx} = 1$
- ▶ $\frac{dy}{dx} = 0$
- ▶ $\frac{dy}{dx} = -1$
- ▶ $\frac{dy}{dx} = \infty$

MTH401 - Differential Equations - Q. No. 2 (M - 2)

Suggestive form of the constant input function for the non homogeneous differential Equation under the method entitled as "Method of the undetermined coefficient" is

- ▶ $f(x) = e^x$
- ▶ $f(x) = a$
- ▶ $f(x) = e^{ax}(A \cos x + B \sin x)$
- ▶ Suggestive form is impossible.

MTH401 - Differential Equations - Q. No. 3 (M - 2)

Which of the following function is linearly dependant to the exponential function

$x e^x$?

- ▶ $-e^x$
- ▶ e^{-x}
- ▶ $x e^x$
- ▶ $-x e^{-x}$

MTH401 - Differential Equations - Q. No. 4 (M - 2)

Eigen values for the system of the differential equations $X' = AX$ are evaluated for the

- ▶ Solution vector X
- ▶ Coefficient matrix A
- ▶ Differentiated solution vector X'
- ▶ Transpose of the Coefficient matrix A

Fundamental set of the solution vectors $X_1, X_2, \dots, X_n$ for any system of the differential equations are obtained by:

MTH401 - Differential Equations - Q. No. 5 (M - 2)

- ▶ Developing the singleton set $\{X\} = \{c_1 X_1, c_2 X_2, \dots, c_n X_n\}$ of the linear combinations of the solution vectors.
- ▶ Taking derivative of the each solution vector and forming the set $\{X'_1, X'_2, \dots, X'_n\}$

- ▶ Taking Integral of the each solution vector and forming the set $\left\{ \int X_1 dx, \int X_2 dx, \dots, \int X_n dx \right\}$
- ▶ Just verifying their linear independence and establishing the set $\{X_1, X_2, \dots, X_n\}$

MTH401 - Differential Equations - Q. No. 6 (M - 5)

For the family with parameter 'a', of rectangular hyperbola $y = a / x$,

find its corresponding orthogonal trajectory and induce the parameter of the new family.

MTH401 - Differential Equations - Q. No. 7 (M - 5)

(a) Justify $D^2 + 9$ whether annihilates the function $f(x) = \sin 3x$ (3)

(b) Evaluate Wronskian of functions

$\sin |x|$ and $|x|$ for $x < 0$ provided that $|x| = -x$ for $x < 0$ (2)

MTH401 - Differential Equations - Q. No. 8 (M - 8)

(a) Determine the interval of convergence of power series $\sum_{k=0}^{\infty} \frac{2^k}{k} x^k$ (5)

(b) If $\sum_{n=0}^{\infty} n c_n x^n$ and $\sum_{n=1}^{\infty} n c_{n-1} x^{n+1}$ are power series solutions for a differential equation then find their sum by introducing a single summation symbol. (3)

MTH401 - Differential Equations - Q. No. 9 (M - 10)

(a) Develop a recurrence relation for the differential equation $y'' - xy = 0$ by applying power series method. (8)

(b) Discuss shortly the linear independence of the power series solutions www.vuzs.net

$$y_1 = A\sqrt{x} \left[2 + \sum_{k=1}^{\infty} \frac{x^k}{(k+1)!5.8.11 \dots (3k+2)} \right]$$

and

$$y_2 = Bx \left[3 + \sum_{k=1}^{\infty} \frac{x^k}{k!1.4.7 \dots (3k-2)} \right]$$



of the differential equation

(2)

MTH401 - Differential Equations - Q. No. 10 , (M - 10) $y'' - \frac{1}{x}y' + \frac{1}{(x-1)^3}y = 0$

(a) Determine the singular points of differential equation classify each singular point as regular or irregular. **5 marks** .Also

(b) Using Rodrigues Formula; $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$ to generate fourth Legendre's polynomial $P_3(x)$. **5 marks**

MTH401 - Differential Equations - Q. No. 11 , (M - 5)

$$\frac{dx}{dt} = 3x - 4y$$

$$\frac{dy}{dt} = 4x - 7y$$

Verify that the vector $X = \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-5t}$ is a solution of the system ;