MIDTERM EXAMINATION

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MTH401- Differential Equations

Total Marks: 50

Duration: 60min

Instructions

1. Attempt all questions.
2. The Time allowed for this paper is 60 minutes.
3. This examination is closed book, closed notes, closed neighbors; any one found cheating will get zero grades in the course MTH401 Differential Equations.
4. You are not allowed to use any type of Table for Formulae of Differentiation and integration during your exam.
5. Each objective type question carries 2 marks and each Descriptive question carries 10 marks. So write every step of the solution of descriptive question to get maximum marks.
6. Do not ask any questions about the contents of this examination from anyone. If you think that there is something wrong with any of the questions, attempt it to the best of your understanding.

Order of the differential equation is the highest order derivative in a differential equation.

T

F

02

e^{-x} Is integrating factor of the differential equation $yx\,y \quad 1 \; dx \quad x \quad 2y\,dy \quad 0$.

T

F

02

If $_1: G(x, y, c_1) \quad 0$ and $_2: H(x, y, c_2) \quad 0$ then each curve of the families is said to be orthogonal intersect all curves of another family orthogonal trajectories of the other.

T

F

02

The given functions $f_1\ x\quad 1\quad x,\ f_2\ x\quad x,\ f_2\ x\quad x^2$ are linearly independent.

T

F

02

A set of functions whose wronskian is zero guarantees that set of functions is linearly dependent.

T

F

02

(a) Define separable form. Just separate the variables of the given differential equation.

$$(r\theta - 4r\quad \theta - 4)dr - \ r^2\theta\quad 20r^2 - \theta - 20\ \ d\theta\quad 0$$

The differential equation of the form d y / d x = f (x, y) is called **separable** if it can be written in the form d y / d x = h(x)g(y)

$$(r\theta - 4r\quad \theta - 4)\,dr - \ r^2\theta\quad 20r^2 - \theta - 20\ \ d\theta\quad 0$$

$$(r\theta - 4r\quad \theta - 4)\,dr\qquad r^2\theta\quad 20r^2 - \theta - 20\ \ d\theta$$

$$r(\theta - 4)\quad 1(\theta - 4)\ \ dr\qquad r^2\ \ \theta\quad 20\ \ -1(\theta\quad 20)\ \ d\theta$$

$$r\quad 1\ \ \theta - 4\ \ dr\qquad r^2 - 1\ [\theta\quad 20\ \ d\theta\ ]$$

$$\frac{r\quad 1}{r^2 - 1}\,dr\qquad \frac{\theta\quad 20}{\theta - 4}\,d\theta$$

$$\frac{1}{r - 1}\,dr\qquad \frac{\theta\quad 20}{\theta - 4}\,d\theta$$

(b) Check whether the given differential equation is exact or not if not then make it exact also show that it is exact.

$$2y^2\quad 3x\ \ dx\quad 2xy\,dy\quad 0$$

(Just make the equation exact do not solve it further)

$$2y^2 \quad 3x\,dx \quad 2xydy \quad 0$$
$$M \quad xy \quad 2y^2 \quad 3x, N\,xy \quad , \quad 2xy$$
$$M_y \quad 4y \quad N_x \quad 2y$$
$$M_y \neq N_x$$

Thus it is not exact now we apply techniques to make it exact

$$2y^2 \quad 3x\,dx \quad 2xydy \quad 0$$
$$\frac{M_y - N_x}{N} \quad \frac{4y-2y}{2xy} \quad \frac{2y}{2xy} \quad \frac{1}{x}$$
$$I.F \quad e^{\int \frac{1}{x}dx} \quad e^{\ln x} \quad x$$
$$2xy^2 \quad 3x^2 \quad dx \quad 2x^2ydy \quad 0$$
$$M \quad xy \quad 2xy^2 \quad 3x^2, \quad N\,xy, \quad 2x^2y$$
$$M_y \quad 4xy \quad N_x \quad 4xy$$
$$M_y \quad N_x$$

Which shows that now equation is exact

(a) Solve given Bernoulli equation $xy - \frac{dy}{dx} \quad y^3e^{-x^2}$ **(Just make the given equation linear in v, do not integrate)**

$$xy - \frac{dy}{dx} \quad y^3e^{-x^2}$$
$$xy^{-2} - \frac{dy}{dx}y^{-3} \quad e^{-x^2}$$
$$put\; y^{-2} \quad v$$
$$-2\frac{dy}{dx}y^{-3} \quad \frac{dv}{dx}$$
$$-\frac{dy}{dx}y^{-3} \quad \frac{1}{2}\frac{dv}{dx}$$
Then
$$\frac{1}{2}\frac{dv}{dx} \quad vx \quad e^{-x^2}$$
$$\frac{dv}{dx} \quad 2vx \quad 2e^{-x^2}$$
Thus it is linear in “v”.

(b)

The population of a town grows at rate proportional to the population at any time. Its initial population of 100 decreased by 20% in 20 years what will be the population in 30 years? **(Just make the model of the population dynamics as well as just describe the given conditions do not solve further)**

Suppose that P_0 is the initial population of the town, as given P_0 is 100 and P(t) the population at any time t then

Population growth by the differential equation

$$\frac{dP}{dt} \quad P$$

$$\frac{dP}{dt} \quad kP$$

$$\frac{dP}{P} \quad kdt$$

Integrate both sides

$$\ln P \quad kt \quad c$$

$$P \quad e^{kt\ c}$$

$$P \quad e^{kt} e^{c}$$

$$P \quad P_0 e^{kt} \qquad say\ P_0 \quad e^{c}$$

Where P_0 is the initial population of the town

P_0 =100 = P(0)

P(20)=100- $\frac{20}{100}$ 100

P(20)=80

P (30) =?

(a) **(also write the formulae)**.

Find a second solution of following differential equations where the first solution is given $x^2 y'' - 5xy' \quad 9y \quad 0; \quad y_1 \quad x^3 \ln x$

$$x^2 y'' - 5xy' \quad 9y \quad 0; \quad y_1 \quad x^3 \ln x$$

differential equation can be written as

$$y'' - \frac{5}{x} xy' \quad \frac{9}{x^2} \quad 0$$

the 2nd solution is given by

$$y_2 \quad y_1 \int \frac{e^{-\int p dx}}{y_1} dx$$

$$y_2 \quad x^3 \ln x \int \frac{e^{-\int \frac{5}{x} dx}}{x^3 \ln x}{}_2\, dx$$

$$y_2 \quad x^3 \ln x \int \frac{e^{\int \frac{5}{x} dx}}{x^3 \ln x}{}_2\, dx$$

$$y_2 \quad x^3 \ln x \int \frac{e^{5 \ln x}}{x^3 \ln x}{}_2\, dx$$

$$y_2 \quad x^3 \ln x \int \frac{e^{\ln x^5}}{x^3 \ln x}{}_2\, dx$$

$$y_2 \quad x^3 \ln x \int \frac{x^5}{x^6 \quad \ln x}{}_2\, dx$$

$$y_2 \quad x^3 \ln x \int \frac{(\ln x)^{-2}}{x} dx$$

$$y_2 \quad x^3 \ln x \frac{(\ln x)^{-21}}{-21}$$

$$y_2 - \quad x^3 \ln x \frac{1}{\ln x}$$

$$y_2 - \quad x^3$$

(b) Solve the differential equation by the undetermined coefficient **(superposition approach)**

$$y'' - 2y' - 3y \quad 2\cos\theta$$

If complimentary solution is given below

$$y_c \quad c_1 e^{3x} \quad c_2 e^{-x}$$

Then just find particular solution.

We find a particular solution of non-homogeneous differential equation.

Suppose input function

$$y_p \quad A\cos\theta \quad B\sin\theta$$

Then

$$y'_p - \quad A\sin\theta \quad B\cos\theta$$

$$y''_p - \quad A\cos\theta - B\sin\theta$$

Substituting in the given differential equation

$$-A\cos\theta - B\sin\theta - 2 \quad -A\sin\theta \quad B\cos\theta \quad -3 \quad A\cos\theta \quad B\sin\theta \qquad 2\cos\theta$$

$$-A \quad 2B - 3A \quad \cos\theta - \quad B \quad 2A - 3B \quad \sin\theta \quad 2\cos\theta$$

From the resulting equations

$-A \quad 2B - 3A \quad 2;$ $\qquad$ $-B \quad 2A - 3B \quad 0$

$-4A - 2B \quad 2;$ $\qquad$ $2A - 4B \quad 0$

$2AB \quad -1;$ $\qquad$ $A B - 2 \quad 0 \rightarrow A \quad 2B$

$\rightarrow 2(2B) \quad B \quad -1$

$\rightarrow 5B \quad -1$

$\rightarrow B \quad \frac{-1}{5}$

$\rightarrow A \quad 2 \frac{-1}{5} \quad \frac{-2}{5}$

$$y_p \quad \frac{-2}{5}\cos\theta \left(\frac{-1}{5} \right) \sin\theta$$

(a) Solve the differential equation by the undetermined coefficient **(annihilator operator).**

$y'' - 4y' \quad x^3 - 2x \quad 1$

If complimentary solution is given below

$y_c \quad c_1 \quad c_2 e^{4x}$

Then find general solution.

In this case of input function is

g(x)= $x^3 - 2x \quad 1$

further

$D^4(g(x)) \quad D^4(x^3 - 2x \quad 1) \; 0$

Therefore the differential operator D^4 annihilates the function g. operating on both sides

$D^4(D^2 - 4D)y \; D^4(x^3 - 2x \quad 1)$

$D^4(D^2 - 4D)y \quad 0$

This is the homogeneous equation of order 5. Next we solve this higher order equation.

Thus auxiliary equation is

$m^4(m^2 - 4m) \quad 0$

$m^5(m - 4) \quad 0$

$m \quad 0,0,0,0,0,4$

Thus its general solution of the differential equation must be

$y \quad c_1 \quad c_2 x \; c_3 x^2 \quad c_4 x^3 \quad c_5 x^4 \quad c_6 e^{4x}$

(b) Solve the differential equation by the variation of parameters

$y'' - 2y' - 3y \quad x^3$

If complimentary solution is given below

$y_c \quad c_1e^{3x} \quad c_2e^{-x}$

Then just find particular solution **(do not integrate)**

$y''-2y'-3y \quad x^3$

This equation is already in standard form

y″+ P(x)y′ +Q(x)y = f(x)

Therefore, we identify the function f(x) as

f(x) = x^3

Since $y_1 \quad e^{3x}, y_2 \quad e^{-x}$ so

We construct the determinants

$$W(y_1, y_2) \quad \begin{vmatrix} e^{3x} & e^{-x} \\ 3e^{3x} & -e^{-x} \end{vmatrix} \quad -e^{3x-x} - 3e^{3x-x} \quad -4e^{2x}$$

$$W_1 \quad \begin{vmatrix} 0 & e^{-x} \\ x^3 & -e^{-x} \end{vmatrix} \quad -x^3e^{-x}$$

$$W_2 \quad \begin{vmatrix} e^{3x} & 0 \\ 3e^{3x} & x^3 \end{vmatrix} \quad x^3e^{3x}$$

We determine the derivatives of the function u_1 and u_2

$$u'_1 \quad \frac{W_1}{W} \quad \frac{-x^3e^{-x}}{-4e^{2x}} \rightarrow u_1 \quad \int \frac{x^3}{4}e^{-3x}dx$$

$$u'_2 \quad \frac{W_2}{W} \quad \frac{x^3e^{3x}}{-4e^{2x}} \rightarrow u_2 \quad \int \frac{x^3}{4}e^{x}dx$$

$$y_p \quad u_1e^{3x} \quad u_2e^{-x}$$

is required particular solution